$$\mathcal{U} = \mathbb{CP}^2 \setminus \{[1:0:0]\}$$

$\{[z_1:z_2] : (z_1,z_2) \neq (0,0)\}$

$$\Sigma = \{[0:z_1:z_2] : (z_1,z_2) \neq (0,0)\} \cong \mathbb{CP}^1 \cong S^2.$$

Want : $\Sigma$ is a deformation retract of $\mathcal{U}$.

$$\psi_t(\underbrace{[z_0:z_1:z_2]}_{\in\, \mathcal{U}}) = [(1-t)z_0 : z_1 : z_2]$$

* need $=0$ when $\boxed{t=1}$

* need $z_0$ when $\boxed{t=0}$

(1) $\psi_0([z_0:z_1:z_2]) = [z_0:z_1:z_2]$ ✓

$\forall\, [z_0:z_1:z_2] \in \mathcal{U}$

(2) $\psi_1(\underbrace{[z_0:z_1:z_2]}_{\neq [1:0:0]}) = [0:z_1:z_2] \in \Sigma$. ✓

$\therefore\ (z_1,z_2) \neq (0,0)$

(3) If $[z_0:z_1:z_2] \in \Sigma$, then $[z_0:z_1:z_2] = [0:z_1:z_2]$

$$\psi_t([\cancel{z_0}:z_1:z_2]) = \psi_t([0:z_1:z_2])$$
$$= [(1-t)0 : z_1 : z_2]$$
$$= [0:z_1:z_2] \qquad \forall\, t \in [0,1]$$

✓

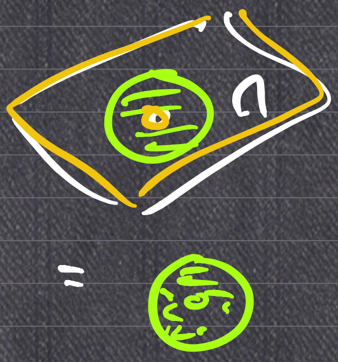
$\mathbb{CP}^2 = \mathcal{U} \cup V$ where :

$\mathcal{U} = \mathbb{CP}^2 \setminus \{[1:0:0]\} \rightsquigarrow \Sigma \cong \mathbb{CP}^1 = S^2$

$V = B^4_\varepsilon([1:0:0])$ open in $\mathbb{CP}^2$.

$U \cap V = B^4_\varepsilon([1:0:0]) \setminus \{[1:0:0]\}$

$\hookrightarrow$ deformation retract onto $S^3$.



(a) $$H^k(S^n) = \begin{cases} 1 & \text{if } k=0 \text{ or } n \\ 0 & \text{if } 0<k<n \\ & \text{or } k>n \end{cases}$$

$$\boxed{0} \to \overset{1}{H^0(\mathbb{CP}^2)} \to \overset{S^2\ (1+1)}{H^0(\cancel{U}) \oplus H^0(V)} \to \overset{1\ \ S^3}{H^0(\cancel{U\cap V})}$$

$$\hookrightarrow \overset{x}{H^1(\mathbb{CP}^2)} \to \boxed{\overset{S^2\ (0+0)}{H^1(\cancel{U}) \oplus \cancel{H^1(V)}}} \to \boxed{\overset{0\ \ S^3}{H^1(\cancel{U\cap V})}}$$

$$\hookrightarrow \overset{y}{H^2(\mathbb{CP}^2)} \to \overset{S^2\ (1+0)}{H^2(\cancel{U}) \oplus \cancel{H^2(V)}} \to \boxed{\overset{0\ \ S^3}{H^2(\cancel{U\cap V})}}$$

$$\hookrightarrow \overset{z}{H^3(\mathbb{CP}^2)} \to \boxed{\overset{S^2\ 0\ +\ 0}{H^3(\cancel{U}) \oplus \cancel{H^3(V)}}} \to \overset{1\ \ S^3}{H^3(\cancel{U\cap V})}$$

$$\hookrightarrow \underbrace{H^4(\mathbb{CP}^2)}_{1} \to \overset{0}{\cancel{H^4(U)}} \oplus \overset{0}{\cancel{H^4(V)}} \to \overset{0\ \ S^3}{H^4(\cancel{U\cap V})}$$

$1-2+1-x=0 \Rightarrow x=0$

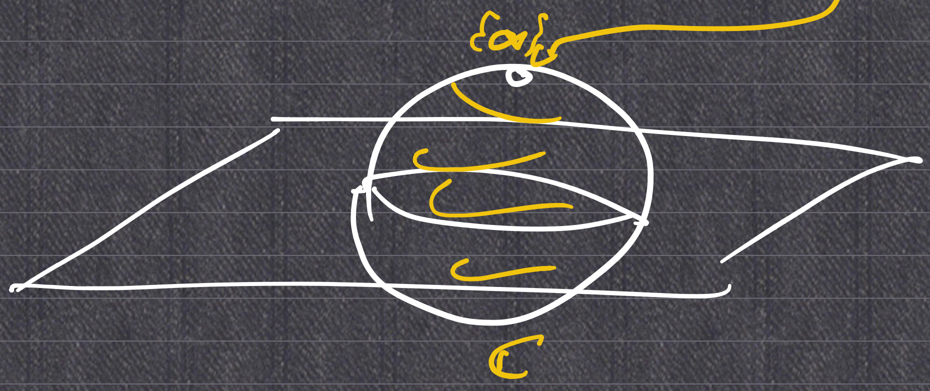
$y-1=0 \Rightarrow y=1$

$z=0$.

$$\dim H^k_{dR}(\mathbb{CP}^2) = \begin{cases} 1 & \text{if } k=0 \\ 0 & \text{if } k=1 \\ 1 & \text{if } k=2 \\ 0 & \text{if } k=3 \\ 1 & \text{if } k=4 \\ 0 & \text{if } k\geq 5. \end{cases}$$

$\mathbb{CP}^1 = \{ [z_0 : z_1] : (z_0, z_1) \neq (0,0) \}.$

$= \underbrace{\{ [z_0 : 1] : z_0 \in \mathbb{C} \}}_{\mathbb{C}} \cup \underbrace{\{ [1:0] \}}_{\{\infty\}}.$



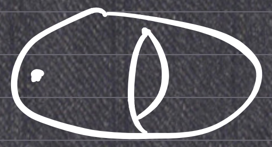

$\Sigma^2 \subset \mathbb{R}^3$ regular surface, compact, $\partial\Sigma = \emptyset$.

Gauss-Bonnet Theorem.

$$\int_{\Sigma^2} K \, d\sigma = 2\pi \chi(\Sigma^2)$$

$K$: Gauss curvature. $d\sigma$: surface form. $\chi$: Euler Characteristics

$\chi = V - E + F.$



$\Phi : \Sigma \to \Phi(\Sigma)$

$\Phi(\Sigma)$ $\quad \displaystyle\int_{\Phi(\Sigma)} K_{\Phi(\Sigma)} \, d\sigma_{\Phi(\Sigma)}$

Chern: $\Phi^*(K_{\Phi(\Sigma)} d\sigma_{\Phi(\Sigma)}) - K_\Sigma d\sigma_\Sigma = d(\ ?\ )$

$$\int_\Sigma \Phi^*(K_{\Phi(\Sigma)} d\sigma_{\Phi(\Sigma)}) - K_\Sigma d\sigma_\Sigma = \int_\Sigma d(\ ?\ )$$

$$= \int_{\partial\Sigma} ? = 0$$

$$\partial\Sigma = \phi.$$

$$\Rightarrow \int_{\Phi(\Sigma)} K_{\Phi(\Sigma)} d\sigma_{\Phi(\Sigma)} - \int_\Sigma K_\Sigma d\sigma = 0.$$

$$\underbrace{c_1(\Sigma)} := \frac{1}{2\pi}[K_\Sigma d\sigma_\Sigma] \in H^2_{dR}(\Sigma)$$

↑

first Chern class