MATH 4033
Calculus on Manifolds
2020-21 Spring

https://canvas.ust.hk/courses/35567

Instructor	**Prof. Frederick Tsz-Ho FONG**
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Office	Room 3488, Department of Mathematics
Class Hours	Tuesday and Thursday 1:30pm-2:50pm (Lectures) - Zoom ID: 918 1296 3896
	Tuesday 6:00pm-6:50pm (Tutorials and Q&A) - Zoom ID: 958 5151 2001

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Course Description

Course outline: It is an advanced undergraduate course about differentiable manifolds, tensor algebra and calculus, and cohomology for motivated and mathematically mature undergraduate majors in mathematics, physics, and related fields. The goal of the course is to equip students with essential knowledge of abstract manifolds for further studies or research in advanced topics in pure mathematics and theoretical physics including Riemannian geometry, Lie theory, general relativity, string theory, etc. Topics will include: regular surfaces in $\mathbb{R}^3$, abstract manifolds, tensors and differential forms, generalized Stokes' Theorem, de Rham cohomology.

Prerequisites: The official prerequisite is MATH 3033 or 3043. Students should have exposure on some basic analysis of multivariable functions (namely, Chapter 13 in MATH 3033, or Chapter 8 in MATH 3043). A solid conceptual understanding of Linear Algebra (MATH 2131 preferred) and Multivariable Calculus is absolutely necessary. Students should be familiar with notions of vector spaces, bases, linear transformations, multivariable chain rule, inverse function theorem, etc.

Credits: 3

Intended Learning Outcomes (ILOs)

Upon completion of this course, students are expected to:

(1) be familiar with the basic notions of differential manifolds;
(2) be equipped with essential background of tensor calculus for further studies in Riemannian geometry, general relativity, and related areas in mathematics and physics;
(3) appreciate the beautiful unification of Green's, Stokes' and Divergence Theorems;
(4) understand the basics of de Rham cohomology.

Student Learning Resources

Major Reference: Lecture Notes written by the instructor

Recommended References:

(1) *Introduction to Smooth Manifolds* by John M. Lee
(2) *Differential Forms and Applications* by Manfredo Do Carmo
(3) *Lectures on Differential Geometry* by S.S. Chern et. al.
(4) *From Calculus to Cohomology* by Ib Madsen and Jorgen Tornehave

Grading

Homework: There will be 4 or 5 problem sets. No late homework would be accepted. Students can form a group of 1 to 2 students to complete an assignment together and submit one copy of solution. All students in the same group will receive the same score. Grouping can be different in different assignments. Students are encouraged to discuss homework problems with other groups, but are required to write up the solutions by their own.

Presentation: The instructor will leave out some proofs (or part of them) in the lecture notes for students to present them. Each student will be randomly assigned one proof in the reference materials, and he/she needs to digest the proof in detail, and video-tape an oral presentation to explain the detail of the proof.

Examinations: There will be a take-home midterm exam, and a sit-in 3-hour final exam arranged by ARRO. The take-home midterm exam must be completed individually without discussion with classmates or any person, and without consultation on any online forum. HKUST Academic Honor Codes will be strictly enforced in both midterm and final exams.

Grading Scheme:

	Percentage	Assessing Course ILOs
Homework	25%	1, 2, 3, 4
Presentation	5%	1, 2, 3, 4
Midterm	20%	1, 2
Final	50%	1, 2, 3, 4
Total	100%	

TENTATIVE SCHEDULE

Week #	Topics
1	regular surfaces in $\mathbb{R}^3$
2	transition maps, tangent maps for regular surfaces
3	abstract manifolds
4	tangent spaces, tangent maps for abstract manifolds
5	inverse function theorem, immersions, submersions
6	cotangent spaces, tensor products
7	wedge products, differential forms
8	exterior derivatives, manifolds with boundary
9	orientations, integrations on manifolds
10	generalized Stokes' Theorem
11	de Rham cohomology groups
12	deformation retracts, Brouwer's fixed-point theorem
13	Mayer-Vietoris sequences, Poincaré duality