Tutorial_11

1. Find the radius of convergence of each of the following power series

   (a) $\sum_{n=1}^{\infty} n^n z^n$

   (b) $\sum_{n=1}^{\infty} n^2 z^n$

   (c) $\sum_{n=1}^{\infty} \frac{3^n}{n!} z^n$

   (d) $\sum_{n=1}^{\infty} \frac{2^n}{n^2} z^n$

   (e) $\sum_{n=1}^{\infty} \frac{n^3}{3^n} z^n$

   (f) $\sum_{n=1}^{\infty} \frac{n^n}{n!} z^n$

2. Supposed that $\sum_{n=0}^{\infty} c_n (x-a)^n$ converges at $x = b$ where $a, b \in \mathbb{R}$, $b < a$, for what value of x the series must converges or converges absolutely?

3. Let $\{c_n\}_{n\in\mathbb{N}}$ be a infinite sequence of integers such that there are infinitely many $n \in \mathbb{N}$ where $c_n \neq 0$

   (a) Show that the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n z^n$ is at most 1

   (b) Give an example of such series which is a Taylor series of a smooth function with radius of convergence 1

   (c) Give an example of such series which is a Taylor series of a smooth function with radius of convergence $\frac{1}{2}$

   (d) Give an example of such series with radius of convergence $\frac{1}{\pi}$