Tutorial_10

1

Let $\{a_n\}$ be the Fibonacci series, i.e. $a_0 = 1, a_1 = 1, a_2 = 3, \cdots a_{n+2} = a_{n+1} + a_n$.
Let $\{b_n\}$ be a series such that $b_n = \frac{1}{a_n}$.
Given $a_k = \frac{1}{\sqrt{5}}(\beta^n - \alpha^n)$ where $\alpha = \frac{1-\sqrt{5}}{2}$ and $\beta = \frac{1+\sqrt{5}}{2}$.
Show that the infinite series $\sum_0^\infty b_n$ converges.

2

Define bijection $\alpha : \mathbb{N}^+ \to \mathbb{N}^+$, $\alpha(n) = \begin{cases} 2k & n = 2k-1 \\ 2k-1 & n = 2k \end{cases}$

Define bijection $\beta : \mathbb{N}^+ \to \mathbb{N}^+$, $\beta(n) = \begin{cases} 2k+1 & n = 3k+1 \\ 4k+2 & n = 3k+2 \\ 4k & n = 3k \end{cases}$

Define bijection $\gamma : \mathbb{N}^+ \to \mathbb{N}^+$, $\gamma(n) = \begin{cases} 2k+1 & n = k^2+1 \\ 2(k^2-k+i-1) & n = k^2+i \quad i \in [2, 3, 4, \cdots, (2k+1)] \end{cases}$

Also define sequences $\{a_k\}, \{b_k\}$ where $a_k = \frac{(-1)^n}{n^2}$ and $b_k = \frac{(-1)^n}{n}$
Determine if the following series converge.

(a) $\sum_{n=1}^\infty a_{\alpha(n)}$
(b) $\sum_{n=1}^\infty a_{\beta(n)}$
(c) $\sum_{n=1}^\infty a_{\gamma(n)}$
(d) $\sum_{n=1}^\infty b_{\alpha(n)}$
(e) $\sum_{n=1}^\infty b_{\beta(n)}$

3

Show that if $f(x) = \sum_{k=0}^\infty \frac{f^{(k)}(0)x^k}{k!}$ $\forall x \in (-c, c)$ then $\sum_{k=0}^\infty \left|\frac{f^{(k)}(0)x^k}{k!}\right| < \infty$ $\forall x \in (-c, c)$.
i.e. The series $\sum_{k=0}^\infty \frac{f^{(k)}(0)x^k}{k!}$ converge absolutely $\forall x \in (-c, c)$