Tutorial 4

1. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function.
   (a) Let $F(x) = \int_0^x f(\frac{t}{2})dt$ and p be a positive number, show that
   $$F(x) = F(x+p)\forall x \in \mathbb{R} \implies f(x) = f(x+\frac{p}{2})\forall x \in \mathbb{R}$$
   (b) Define $f_{i+1}(x) = \int_0^x f_i(\frac{t}{2})dt$ where $f_0(x)$ is continuous.
   If $f_i(x) = f_i(x+1)$ show that $f_i(x) = 0 \quad \forall x \in \mathbb{R}, i \in \mathbb{N}$

2. (a) Let $f : \mathbb{R} \to \mathbb{R}$ be an integrable odd function i.e. $f(x) = -f(x)$,
   show that $\int_{-a}^{a} f(x)dx = 0$ for any $a \in \mathbb{R}^+$.
   (b) Let $g : \mathbb{R} \to \mathbb{R}$ be an integrable periodic function i.e. $f(x+T) = f(x)$,
   show that $\int_b^{b+T} f(x)dx = \int_0^T f(x)dx$ for any $b \in \mathbb{R}$.
   (c) Let $g : \mathbb{R} \to \mathbb{R}$ be an integrable odd periodic function with period T,
   show that $\int_b^{b+T} f(x)dx = 0$ for any $b \in \mathbb{R}$

3. Let $I_n(x) = \int_0^x e^{mt} \sin^n(t)dt$
   (a) Find the reduction formula for I_n
   (b) Find $I_n(\pi)$