Tutorial 3

Q1:

■ **Exercise 4.26 — Source: HKAL 1994.** Let $f(x) = \int_1^x \sin(\cos t)\, dt$.

(a) Show that f is injective on $[0, \pi/2)$.

(b) Find $\dfrac{d}{dx} f^{-1}(x)\Big|_{x=0}$

Q2:

■ **Exercise 4.27 — Source: HKAL 1997.** Evaluate

$$\lim_{x \to 0+} \left(\frac{1}{x^3} \int_0^x e^{t^2}\, dt - \frac{1}{x^2} \right).$$

Q3:

■ **Exercise 4.34** Prove that integration formula below:

$$\int \frac{1}{\sqrt{x^2 + a^2}}\, dx = \log \left| x + \sqrt{x^2 + a^2} \right| + C$$

$$\int \frac{1}{x^2 - a^2}\, dx = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + C$$

Q4:

■ **Exercise 4.35** Show that for any $r > 0$, we have

$$\int_0^r \sqrt{r^2 - x^2}\, dx = \frac{\pi r^2}{4}.$$

This shows the area of the circle with radius r is πr^2.

Q5:

■ **Exercise 4.32** Compute the following integrals:

1. $\int_a^b x \cos(x^2 + 1)\, dx$
2. $\int_a^b x^3 e^{x^4}\, dx$
3. $\int_a^b \dfrac{x}{1 + x^2}\, dx$

Q6:

■ **Exercise 4.33** Prove that

$$\int \cot x \, dx = \log |\sin x| + C$$

$$\int \csc x \, dx = -\log |\csc x + \cot x| + C$$