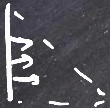
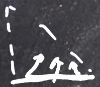
Lecture 19
23/04/2020

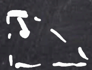
$$\begin{cases} x' = -x + ay + x^2y \\ y' = b - ay - x^2y \end{cases} \qquad \leftarrow F(x,y)$$

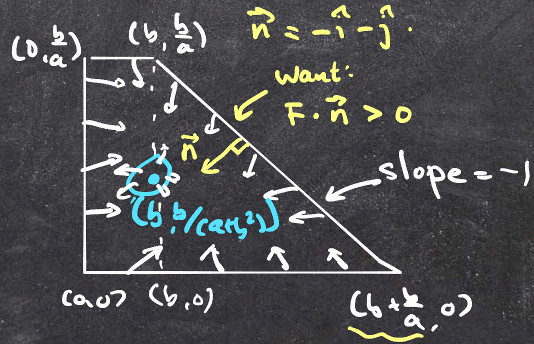
$(a, b > 0).$

 : $F(0,y) = (\underbrace{ay}_{+},\ b - ay)$

 : $F(x,0) = (-x,\ \underbrace{b}_{+})$

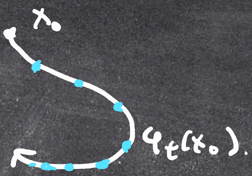
 : $\vec{F}\cdot(-\hat{i}-\hat{j}) = x - ay - x^2y - b + ay + x^2y = x - b > 0 \iff x > b.$

 : $\vec{F}(x, \frac{b}{a}) = \left(-x + b + x^2\cdot\frac{b}{a},\ \cancel{b} - \cancel{b} - \underbrace{x^2\cdot\frac{b}{a}}_{\ominus}\right)$



K = 

$\vec{x}' = \vec{F}(x)$, φ_t flow map.

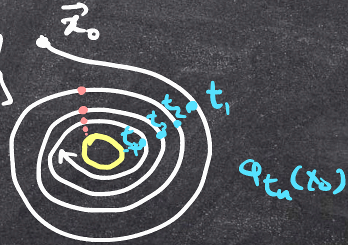


$\vec{x}_0 \in \mathbb{R}^d$

$\varphi_t(\vec{x}_0)$, $t \geq 0$.

$$\underbrace{\omega(\vec{x}_0)}_{} := \left\{ \vec{y}_0 \in \mathbb{R}^d : \exists\, t_n \to +\infty \text{ s.t. } \lim_{n\to\infty} \varphi_{t_n}(\vec{x}_0) = \vec{y}_0 \right\}$$

ω - limit set



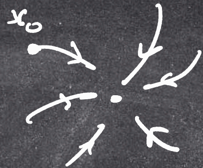
$\alpha(\vec{x}_0)$: $+\infty \rightsquigarrow -\infty$.

$\uparrow$

α - limit set

- $\vec{x}' = A\vec{x}$

eigenvalues of A negative.



$$\varphi_t(x_0) \longrightarrow \vec{0}.$$

$$\lim_{n\to\infty} \varphi_{t_n}(x_0) = \vec{0}.$$

$$\forall t_n \to +\infty.$$

$$\Rightarrow \quad \omega(\vec{x}_0) = \{\vec{0}\}.$$

any point

$$\alpha(\vec{x}_0) = \phi.$$

$\vec{x}_0 \neq \vec{0}$.

$$|\varphi_t(x_0)| \to +\infty$$

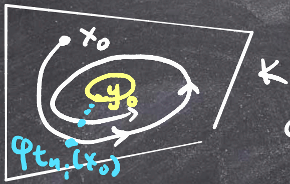
as $t \to -\infty$.

$$\alpha(\vec{0}) = \{\vec{0}\}.$$

$$\phi \neq \{\phi\}.$$

- Key idea of the proof of Poincaré - Bendixison :

To show $\{\varphi_t(y_0)\}$ is periodic



$\varphi_t(x_0) \in K \quad \forall t \geq 0.$

$\uparrow$

compact.

$\forall t_n \to +\infty, \quad \varphi_{t_n}(x_0) \in K$

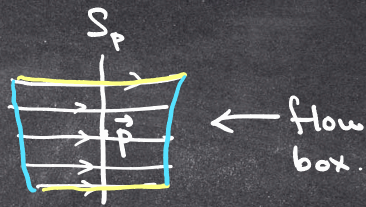
$\Downarrow$ BW

$\exists t_{n_i} \to +\infty$ s.t. $\varphi_{t_{n_i}}(x_0) \xrightarrow{i \to +\infty} y_0 \in K.$

$\Bigg\} \to \underbrace{\omega(x_0)}_{\substack{\ni \\ y_0}} \neq \emptyset$

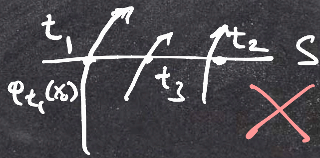
local section, flow box.

$F(\vec{p}) \neq \vec{0}$.

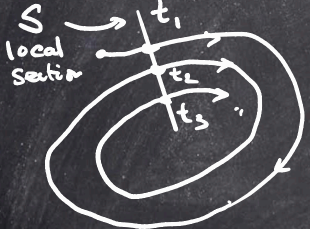


$\vec{F}(\vec{x}) \cdot F(\vec{p}) > 0$.
when $\vec{x} \approx \vec{p}$.

$t_1 < t_2 < t_3$



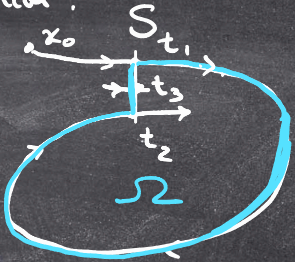
Lemma 4.17



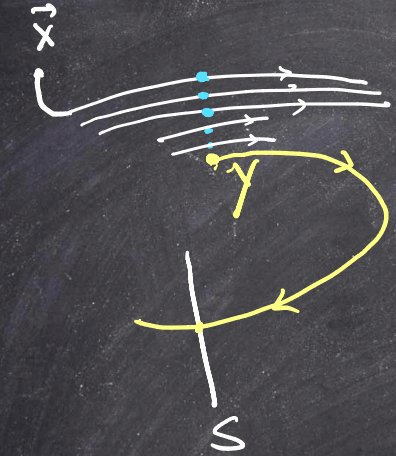
$\forall$ local section S, any trajectory must intersect S in <u>monotonic</u> order.

Proof: By contradiction.

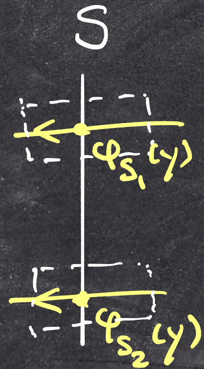
Assume



Lemma 4.18 If $\omega(\vec{x}) \ni \vec{y}$ then $\varphi_t(\vec{y})$ intersects any local section at <u>not</u> <u>more</u> <u>one</u> <u>point</u>.



<u>Proof</u>:



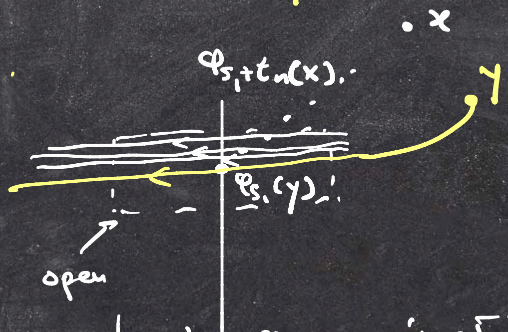
Assume:

$$y = \lim_{n\to\infty} \varphi_{t_n}(x) \qquad (y \in \omega(x))$$

$$\varphi_{s_1}(y) = \lim_{n\to\infty} \varphi_{s_1}\left(\varphi_{t_n}(x)\right) \qquad (\varphi_{s_1} \text{ is cont.})$$

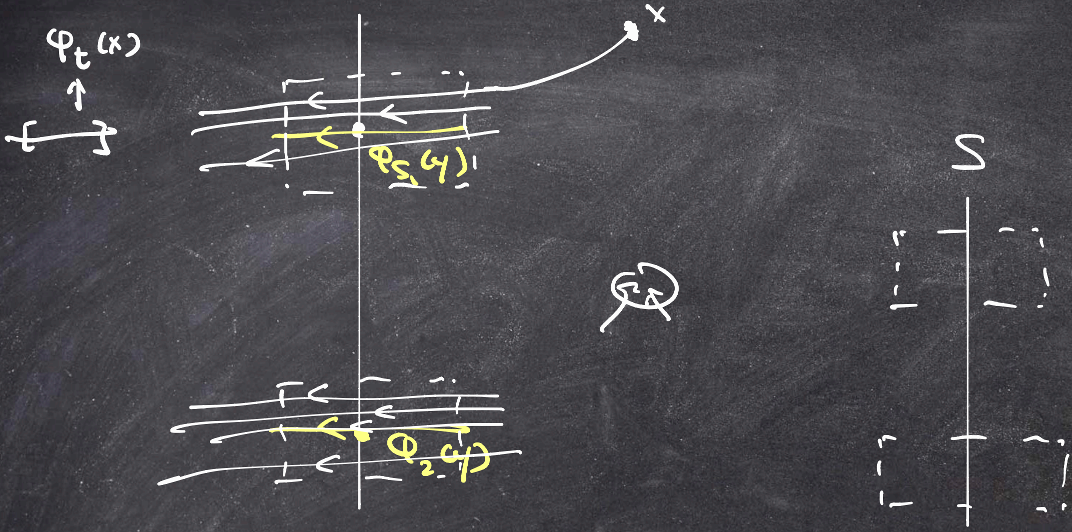
$$= \lim_{n\to\infty} \varphi_{s_1+t_n}(x) \quad \Longrightarrow \quad \varphi_{s_1}(y) \in \omega(x).$$

$$\varphi_{s_2}(y) = \lim_{n\to\infty} \varphi_{s_2+t_n}(x).$$



$\varphi_{s_1+t_n}(x) \to \varphi_{s_1}(y) \Longrightarrow \exists$ infinitely $\varphi_{s_1+t_n}(x)$'s $\in \boxed{}$

$\therefore \Longrightarrow \varphi_t(x)$ pass through S for infinity times.


$\varphi_t(x)$
x
$\varphi_{s_1}(y)$
$\varphi_2(y)$
S