COMP4901K and MATH4824B Final Exam

December 12, 2018, Wednesday

Time: 8:30am-10:30am

Instructor: Yangqiu Song

Name: ________________ **Student ID:** ________________

Question	**Score**	**Question**	**Score**
1	/ 10	6	/ 10
2	/ 5	7	/ 10
3	/ 15	8	/ 10
4	/ 10	9	/ 10
5	/ 10	10	/ 10
Total:	**/ 100**		

Q1. Yes/No Questions (10 Points)

Indicate whether each statement is true (✓) or false ($\times$).

1. For a fully connected deep network with one hidden layer, increasing the number of hidden units could decrease bias and increase variance of the prediction.
2. Recurrent neural networks can handle a sequence of arbitrary length, while feedforward neural networks can not.
3. Multi-layer neural network model trained using stochastic gradient descent on the same dataset with different initializations for its parameters is guaranteed to learn the same parameters.
4. Stochastic gradient descent results in a smoother convergence plot (loss vs epochs) as compared to batch gradient descent.
5. In supervised learning, training data includes both inputs and desired outputs.

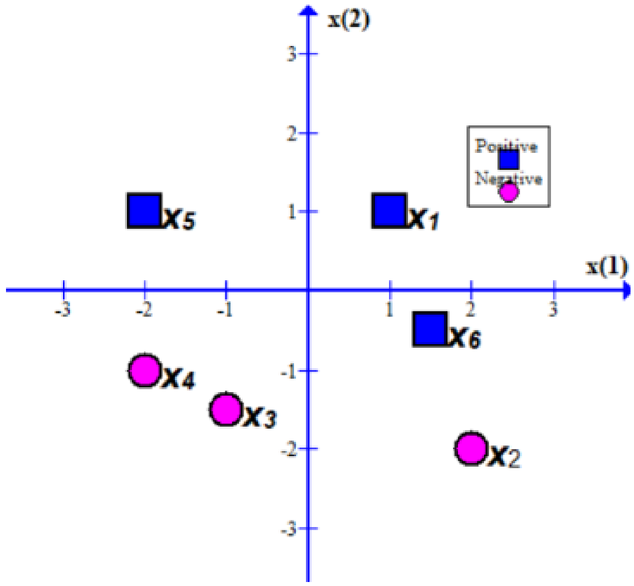
Q2. Short-Answer Questions (5 Points)

Please provide short answers to following questions:

1. What are epoch, batch size, and learning rate in the context of machine learning?
2. What's the risk with tuning hyper-parameters using the test dataset?
3. Provide a typical goal of (good) weight initialization for deep neural networks.

Q3. Perceptron (15 Points)

The perceptron algorithm is to predict the label $y_i' = sgn(w^T x_i)$ for each data point x_i. For the following dataset, please derive the perceptron algorithm and give the solution to w.



We set: learning rate α=0.2 and initial weight vector $w_0 = [1, 0.5, 0]^T$. The input sequence is:

$$x_1 = [1, 1, 1]^T$$
$$x_2 = [2, -2, 1]^T$$
$$x_3 = [-1, -1.5, 1]^T$$
$$x_4 = [-2, -1, 1]^T$$
$$x_5 = [-2, 1, 1]^T$$
$$x_6 = [1.5, -0.5, 1]^T$$

Q4. Logistic Regression (10 Points)

Remember the form of Logistic Regression function

$$P(y = 1|x; \theta) = \frac{1}{1 + e^{-\theta^T x}}$$

Let us assume

$$P(y = 1|x; \theta) = \sigma_\theta(x) \quad \text{and} \quad P(y = 0|x; \theta) = 1 - \sigma_\theta(x)$$

then

$$P(y|x; \theta) = (\sigma_\theta(x))^y (1 - \sigma_\theta(x))^{1-y}$$

Now you can learn your logistic model using maximum likelihood estimation, for which you should use gradient descent to maximize the log-likelihood function iteratively. Derive the **stochastic gradient descent** update rule for logistic regression.

Q5. Statistical Language Models (10 Points)

We build a uni-gram language model based on the following word frequency table. (**Do NOT use any smoothing technique.**)

word	frequency	word	frequency
i	100	hate	50
you	100	book	100
he	50	desk	50
she	50	paper	200
like	100	reading	60
love	100	writing	40

(1) What is the likelihood of the sequence “i love reading” based on the language model you build?

(2) List a disadvantage of the uni-gram language model. (4 points)

Q6. Language Model Applications (10 Points)

Nathan L. Pedant would like to build a spelling corrector focused on the particular problem of there vs. their. The idea is to build a model that takes a sentence as input, for example

He saw their football in the park (1)

He saw their was a football in the park (2)

and for each instance of their or there predict whether the true spelling should be their or there. So for sentence (1) the model should predict their, and for sentence (2) the model should predict there. Note that for the second example the model would correct the spelling mistake in the sentence.

Nathan decides to use a language model for this task. Given a language model $p(w_1, \ldots, w_n)$, he returns the spelling that gives the highest probability under the language model. So for example for the second sentence we would implement the rule

If

$$p(He\ saw\ their\ was\ a\ football\ in\ the\ park) > p(He\ saw\ there\ was\ a\ football\ in\ the\ park)$$

Then Return their

Else Return there.

(1): The first language model Nathan designs is of the form

$$p(w_1, \ldots, w_n) = \prod_{i=1}^{n} q(w_i),$$

where

$$q(w_i) = Count(w_i)/N$$

and $Count(w_i)$ is the number of times that word w_i is seen in the corpus, and N is the total number of words in the corpus. Let's assume

$$N = 10,000,$$

$$Count(there) = 50,$$

and

$$Count(their) = 100.$$

Assume in addition that for every word v in the vocabulary, $Count(v) > 0$. What does the rule given above return for "He saw their was a football in the park?" (there or their?)

(2): The second language model Nathan designs is of the form

$$p(w_1, \ldots, w_n) = q(w_1) \prod_{i=2}^{n} q(w_i|w_{i-1}),$$

where

$$q(w_i|w_{i-1}) = Count(w_i, w_{i-1})/Count(w_{i-1})$$

and $Count(w_{i-1})$ is the number of times that word w_{i-1} is seen in the corpus, and $Count(w_i, w_{i-1})$ is the number of co-occurrences of two words in the corpus. Let's assume

$$Count(there, saw) = 20,$$

$$Count(their, saw) = 40,$$

$$Count(saw) = 200,$$

$$Count(was, there) = 20,$$

$$Count(was, their) = 2,$$

$$Count(there) = 50,$$

$$Count(their) = 100.$$

We also assumes all other probabilities are non-zero. What does the rule given above return for "He saw their was a football in the park?" (there or their?)

Q7. Convolutional Neural Network (10 Points)

A convolutional neural network has one `2x3` convolutional layer with stride `1` and a max pooling layer. Suppose the input sequence is "a cat sits on the mat"; the filter K is defined as follows,

$$K = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

and embeddings (2 dimensional) of input sequence are listed as follows,

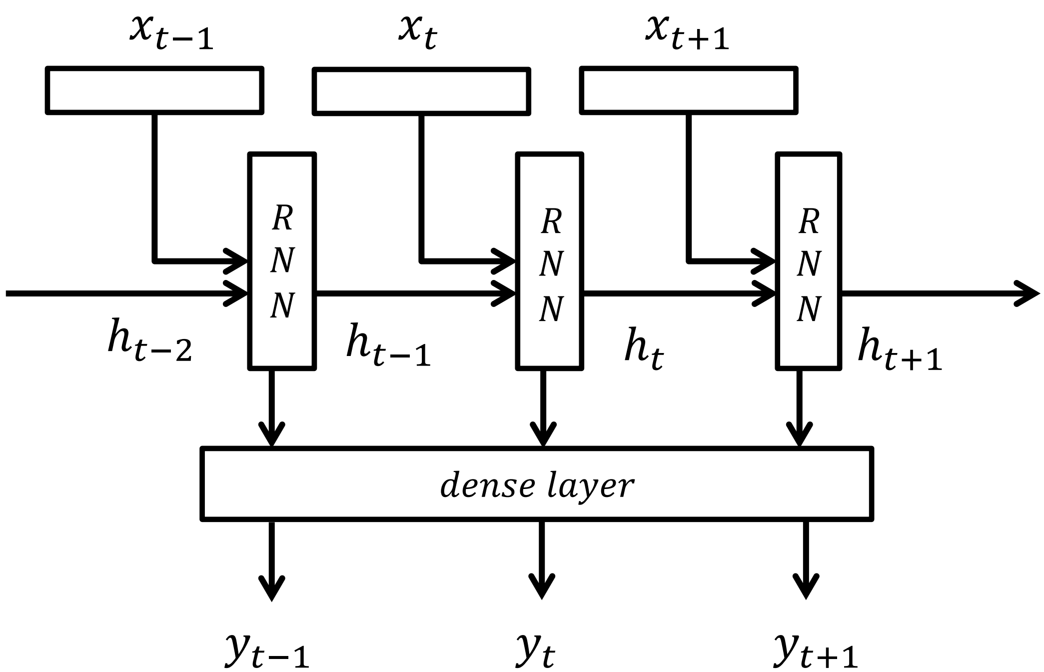
a	cat	sits	on	the	mat
$\begin{bmatrix} 0.1 \\ 1.2 \end{bmatrix}$	$\begin{bmatrix} 0.2 \\ 1.0 \end{bmatrix}$	$\begin{bmatrix} 0.3 \\ 0.8 \end{bmatrix}$	$\begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}$	$\begin{bmatrix} 0.5 \\ 0.4 \end{bmatrix}$	$\begin{bmatrix} 0.6 \\ 0.2 \end{bmatrix}$

(i) What is the output of the max pooling layer? (6 points)
0.6, 0.8, 1.0, 1.2 worth 1 point. 1.2 worth 2 point.

(ii) What is the role of a pooling layer in a convolutional neural network (4 points)?

Q8. Recurrent Neural Network (10 Points)

Considering the following architecture: This architecture can be formalized as follows:

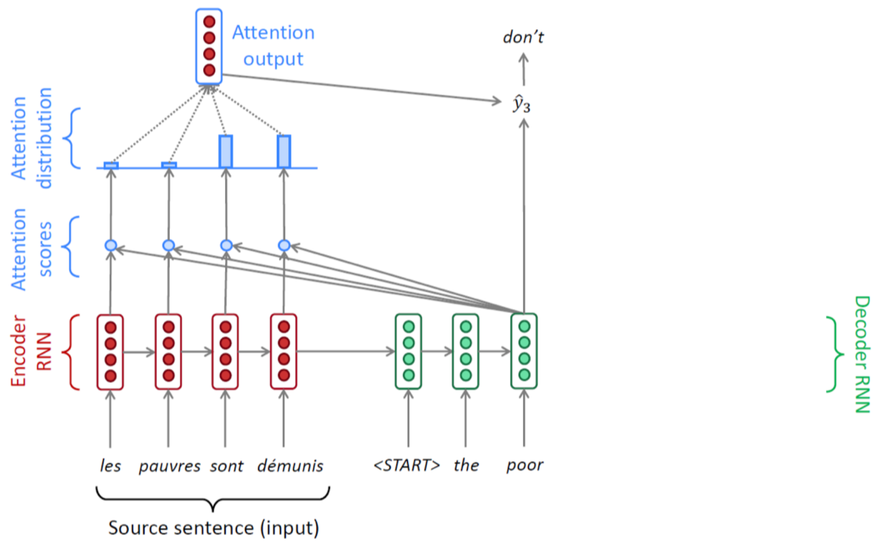


$$h_t = tanh\left(W_h h_{t-1} + W_i x_t\right)$$
$$v_t = relu(W_v h_t + b)$$
$$y_t = \text{softmax}\left(W_o v_t\right)$$

where W_h, W_i, W_v, W_o are the parameters, and $x_t \in R^D$, $h_t \in R^H$, $v_t \in R^V$, $y_t \in R^Y$. Now given $D = 50, H = 100, V = 10, Y = 1000$. Please compute the number of all the parameters in these network in two cases: word embeddings x_t are trainable and not trainable (*hint:* here parameters are those updated while training).

Q9. Dot-Product Attention (10 Points)

Sequence to Sequence (Seq2Seq) is about training models to convert sequences from one domain (e.g. sentences in French) to sequences in another domain (e.g. the same sentences translated to English), introduced in 2014 by Sutskever et al. But the simple version Seq2Seq utilizes limited information. Attention mechanism is a simple but effective way to improve the performance of Seq2Seq.



The details of Dot-product Attention are as follows:

1. Compute the dot product: $e_{t,j} = \boldsymbol{s_{t-1}^T h_j}$, where $\boldsymbol{s_{t-1}^T}$ is the transpose of $\boldsymbol{s_{t-1}}$ which is the decoder hidden state at time $t-1$, $\boldsymbol{h_j}$ is the encoder hidden state at time j.
2. Compute the weight: $\alpha_{t,j} = \frac{\exp(e_{t,j})}{\sum_{k=0}^{T-1}\exp(e_{t,k})}$
3. Compute the attention output: $\boldsymbol{c_t} = \sum_{j=0}^{T-1} \alpha_{t,j}\boldsymbol{h_j}$
4. Concatenate the attention output with decoder hidden state before sending the decoder RNN, or concatenate the attention output with the decoder output (we use the latter method in lab 10).

According to the description above, please compute the attention output at time 2.

h_0	h_1	h_2	h_3	s_0	s_1
$\begin{bmatrix}0.1\\0.2\\0.3\\0.4\end{bmatrix}$	$\begin{bmatrix}0.2\\0.3\\0.1\\0.4\end{bmatrix}$	$\begin{bmatrix}0.1\\0.4\\0.3\\0.2\end{bmatrix}$	$\begin{bmatrix}0.3\\0.4\\0.2\\0.1\end{bmatrix}$	$\begin{bmatrix}0.4\\0.1\\0.2\\0.3\end{bmatrix}$	$\begin{bmatrix}0.2\\0.4\\0.3\\0.1\end{bmatrix}$

Q10. Attention Variants (10 Points)

We have introduced the dot-product attention in the previous question, and there are other attention variants. Please write down the $e_{t,j}$ formulas according to the Keras code below.

<table>
<tr><td>Dot-Product</td><td><pre>weight = Activation('softmax')(
    dot([decoder_outputs[-1], encoder_output],
        axes=[2, 2]))</pre></td><td>$e_{t,j} = \boldsymbol{s_{t-1}^T h_j}$</td></tr>
<tr><td>Multiplicative</td><td><pre>encoder_output_maped = Dense(units=2*hidden_dim,
                             bias=False)(encoder_output)
weight = Activation('softmax')(
    dot([decoder_outputs[-1], encoder_output_maped],
        axes=[2, 2]))</pre></td><td></td></tr>
<tr><td>Additive</td><td><pre>decoder_output_maped = Dense(units=2*hidden_dim,
                             bias=False)(decoder_output)
encoder_output_maped = Dense(units=2*hidden_dim,
                             bias=False)(encoder_output)
tanh_result = Activation('tanh')(Add()([
    Lambda(lambda x: K.expand_dims(x, axis=2))(
        decoder_output_maped),
    Lambda(lambda x: K.expand_dims(x, axis=1))(
        encoder_output_maped)]))
weight = Activation('softmax')(
    Lambda(lambda x: K.squeeze(x, axis=3))(
        Dense(units=1, bias=False)(tanh_result)))</pre></td><td></td></tr>
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