MATH 4033
Calculus on Manifolds
2017-18 Spring
https://canvas.ust.hk/courses/16957

Lectures and Tutorials

Instructor	**Prof. Frederick Tsz-Ho FONG**
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Office	Room 3488, Department of Mathematics
Class Hours and Venue	Tuesday 10:30-11:50 @ Room 5508
	Wednesday, 18:00-18:50 @ Room 5508
	Thursday, 10:30-11:50 @ Room 5508

Course Description

Course outline: This course introduces *differentiable manifolds*, *tensor calculus*, and *cohomology* to motivated undergraduate mathematics and/or physics students. It presents fundamental concepts about abstract manifolds which are essential for further studies on Riemannian geometry, general relativity, string theory, etc. Topics will include: regular surfaces in $\mathbb{R}^3$, abstract manifolds, tensors and differential forms, Lie derivatives, generalized Stokes' Theorem, de Rham cohomology.

Prerequisites: The official prerequisite is MATH 3033 or 3043. The more crucial prerequisites are Linear Algebra (MATH 2131 preferred) and Multivariable Calculus (MATH 2023). Students should have workable background on vector spaces, dimensions and bases, kernels and images, linear transformations, multivariable chain rule, inverse function theorem, vector calculus, etc.

Intended Learning Outcomes (ILOs)

Upon completion of this course, students are expected to:

(1) learn the basic concepts of differentiable manifolds and tensor calculus;
(2) acquire essential knowledge of tensor calculus for further studies in Riemannian geometry, general relativity, string theory, and related areas in mathematics and theoreatical physics;
(3) appreciate the unification of Green's, Stokes' and Divergence Theorems; and
(4) understand the basics of de Rham cohomology.

Student Learning Resources

All course materials will be posted on the Canvas website. The link can be found at the top of this page.

Major Reference: Lecture Notes written by the instructor

Recommended References:

(1) *Introduction to Smooth Manifolds* by John M. Lee
(2) *Differential Forms and Applications* by Manfredo Do Carmo
(3) *A Comprehensive Introduction to Differential Geometry, Vol. 1* by Michael Spivak
(4) *Lectures on Differential Geometry* by S.S. Chern et. al.

Grading

Homework: There will be five problem sets but they are not compulsory. Students are welcome to discuss homework problems with the instructor.

Presentation: Every student/group will give an oral presentation (around 20 to 30 minutes each group) on the proof of a theorem in the lecture notes or in supplementary materials. Depending on class enrollment after the add-drop period, presentations will be either individual or in group.

Examinations: There will be a take-home midterm (dates to be decided and will be after Chapter 3 is covered), and also a 3-hour sit-in, open-notes final exam. Both midterm and final must be completed individually without discussion with classmates or any person. HKUST Academic Honor Codes will be strictly enforced.

Grading Scheme:

	Percentage	Assessing Course ILOs
Homework	0%	1, 2, 3, 4
Presentation	10%	1, 2, 3, 4
Midterm	30%	1, 2, 3, 4
Final	60%	1, 2, 3, 4
Total	100%	

Tentative Schedule

Week #	Topics
1	regular surfaces in $\mathbb{R}^3$
2	transition maps, tangent maps for regular surfaces
3	abstract manifolds
4	tangent spaces, tangent maps for abstract manifolds
5	inverse function theorem, immersions, submersions
6	cotangent spaces, Lie derivatives, tensor products
7	wedge products, differential forms
8	exterior derivatives, manifolds with boundary
9	orientations, integrations on manifolds
10	generalized Stokes' Theorem
11	de Rham cohomology, deformation retracts
12	Mayer-Vietoris sequences
13	selected topics