Statistical Learning for Text Data Analytics
Word Embeddings

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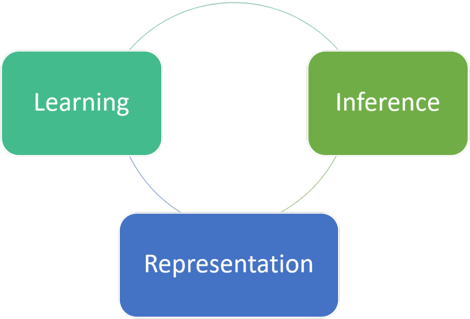
$*$Contents are based on materials created by Noah Smith, Richard Socher, Percy Liang, Hongning Wang, David Jurgens, Mohammad Taher Pilehvar, Maneesh Sahani

Reference Content

- Noah Smith. CSE 517: Natural Language Processing https://courses.cs.washington.edu/courses/cse517/16wi/
- Richard Socher. CS224d: Deep Learning for Natural Language Processing. https://web.stanford.edu/class/cs224d/
- Percy Liang. ICML tutorial on Natural Language Understanding: Foundations and State-of-the-Art https://icml.cc/2015/tutorials/icml2015-nlu-tutorial.pdf
- Hongning Wang. CS6501 Text Mining. http://www.cs.virginia.edu/~hw5x/Course/Text-Mining-2015-Spring/_site/
- David Jurgens and Mohammad Taher Pilehvar. EMNLP 2015 Tutorial – Semantic Similarity Frontiers: From Concepts to Documents. http://www.emnlp2015.org/tutorials/34/34_OptionalAttachment.pdf
- Maneesh Sahani. Dimensionality Reduction. http://www.gatsby.ucl.ac.uk/~maneesh/dimred/dimred.pdf

Course Topics



- Representation: language models, word embeddings, topic models
- Learning: supervised learning, semi-supervised learning, sequence models, deep learning, optimization techniques
- Inference: constraint modeling, joint inference, search algorithms

NLP applications: tasks introduced in Lecture 1

Overview

1. Overview
   - Language Models: Recap
   - Vector Space Model
2. Word Embeddings
   - Efficiency: Hierarchical Softmax
   - Efficiency: Negative Sampling
   - Evaluation

Paragraphs of Text

- A language model is a probability distribution over $\mathcal{V}^\dagger$
- Typically P decomposes into probabilities $P(x_i|\mathbf{h}_i)$
  - We considered n-gram, log-linear, and neural language models, etc.
- Today: probabilistic models that relate a word and its cotext (the linguistic environment of the word)
- This might help us learn to represent words, contexts, or both

Three Kinds of Cotext

If we consider a word token at a particular position i in text to be the observed value of a random variable X_i, what other random variables are predictive of/related to X_i?

- The words that occur within a small "window" around i (e.g., $x_{i-2}, x_{i-1}, x_{i+1}, x_{i+2}$, or maybe the sentence containing i) $\rightarrow$ distributional semantics
- The document containing i (a moderate-to-large collection of other words) $\rightarrow$ topic models
- A sentence known to be a translation of the one containing i $\rightarrow$ translation models

Overview

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Context of Words

Example

Let's try to keep the kitchen ______.

Example

We used log-linear model to ______ the test data set.

What does ______ mean?

Let's try to keep the kitchen ____.

- Observation: context can tell us a lot about word meaning
- Context: local window around a word occurrence (for now)
- Roots in linguistics:
  - Distributional hypothesis: Semantically similar words occur in similar contexts (Harris (1954))
  - "You shall know a word by the company it keeps." (Firth (1957))
- Pros: data-driven, easy to implement
- Cons: ambiguity

Distributional Semantics

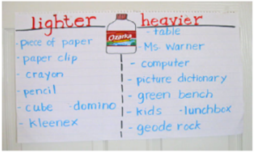
- The distributional hypothesis in linguisticsis derived from the semantic theory of language usage, i.e. words that are used and occur in the same contexts tend to purport similar meanings
- The basic idea of distributional semantics can be summed up in the so-called distributional hypothesis: linguistic items with similar distributions have similar meanings

We will mention distributed representation based on neural network models later

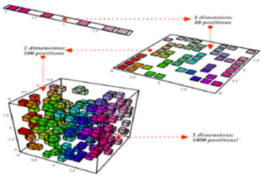
Corpus based Approach

1) Corpus

2) Preprocessing



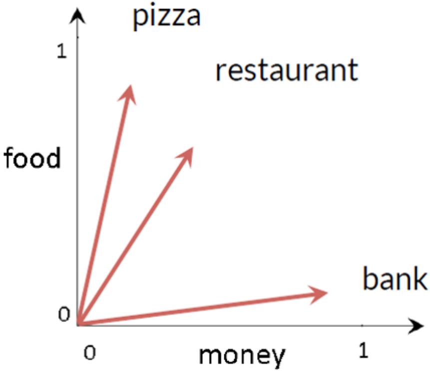
3) Dimensionality Reduction



4) Post Processing

Vector Space Model (VSM)

- Represent each word with its context words



Local Contexts: Distributional Semantics

- Within NLP, emphasis has shifted from topics to the relationship between $v \in \mathcal{V}$ and more local contexts
- These models are designed to "guess" a word at position i given a word at a position in $[i-c, i-1] \cup [i+1, i+c]$
- Sometimes such methods are used to "pre-train" word vectors used in other, richer models (like neural language models)

Context Vector Construction

- Form a word-context matrix of counts (data)

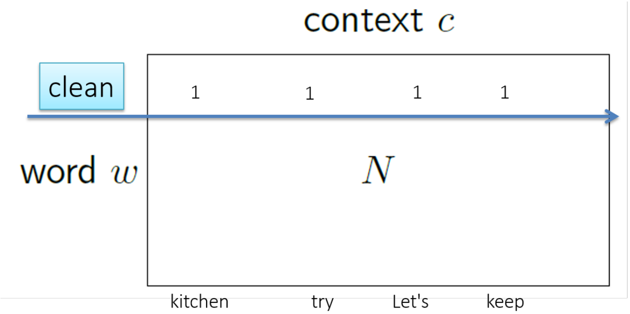


Figure: "Let's try to keep the kitchen clean."

Context Vector Construction

- Words on left, words on right

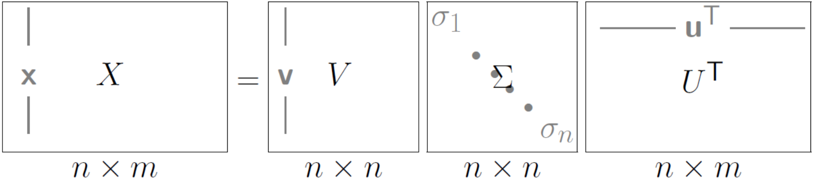
	cats_L	dogs_L	tails_R	have_L	have_R
cats	0	0	0	0	1
dogs	0	0	0	0	1
have	1	1	1	0	0
tails	0	0	0	1	0

Figure: "Doc1: Cats have tails. Doc2: Dogs have tails."

- Usually used for part-of-speech induction

Singular Value Decomposition (SVD):

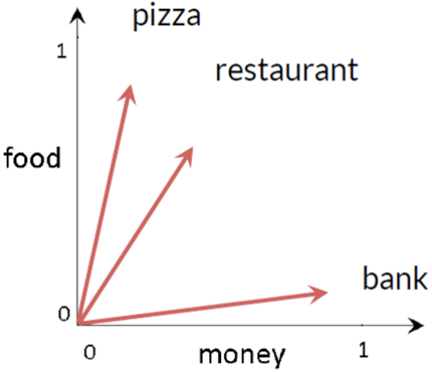
- Let $\mathbf{X} = (\mathbf{x}_1, \ldots, \mathbf{x}_m)$, where $\mathbf{x}_i \in \mathbb{R}^n$, so $\mathbf{X} \in \mathbb{R}^{n \times m}$
- SVD computes $\mathbf{X} = \mathbf{V}\Sigma\mathbf{U}^\top$ with
  - $\mathbf{V}\mathbf{V}^\top = \mathbf{V}^\top\mathbf{V} = \mathbf{I}$, the orthonormal basis $\{\mathbf{v}_i\}$ for the columns of $\mathbf{X}$
  - $\mathbf{U}\mathbf{U}^\top = \mathbf{V}^\top\mathbf{V} = \mathbf{I}$, the orthonormal basis $\{\mathbf{u}_i\}$ for the rows of $\mathbf{X}$
  - Σ is a diagonal matrix containing singular values in decreasing order $\sigma_1 \geq \sigma_2 > \cdots > \sigma_n$ (if $n < M$)



Truncated at k: Approximating $\mathbf{X}$ by truncating $\sigma_i < \theta$ equates to a "low rank approximation"

Similarity between Words

$$cos(\theta) = \frac{\mathbf{a}^\top \mathbf{b}}{\|\mathbf{a}\|_2 \|\mathbf{b}\|_2}$$



Overview

Problems with SVD

- Computational cost scales quadratically for $n \times m$ matrix: $O(mn^2)$ flops (when $n < m$)
  - Could be less when the matrix is sparse, but still very inefficient in practice
  - Bad for millions of words or documents
- Hard to incorporate new words or documents
- Different learning regime than other DL models

Idea: Directly Learn Low-dimensional Word Vectors

- Old idea
  - Learning representations by back-propagating errors (Rumelhart et al. (1986))
  - A neural probabilistic language model (Bengio et al. (2003)
  - NLP (almost) from Scratch (Collobert et al. (2011))
- A recent, even simpler and faster model: word2vec
- Usually called distributed representations in the context of deep learning
  - Vector representation does not represent a distribution, but distributed over the space
  - Term widely used in connectionism (Hinton (1986)):
    - "In the componential approach each concept is simply a set of features and so a neural net can be made to implement a set of concepts by assigning a unit to each feature and setting the strengths of the connections between units so that each concept corresponds to a stable pattern of activity distributed over the whole network."

Main Idea of Word2vec

Mikolov et al. (2013a,b)

- Instead of capturing co-occurrence counts directly
- Predict surrounding words of every word
- Faster and can easily incorporate a new sentence/document or add a word to the vocabulary

Word2vec

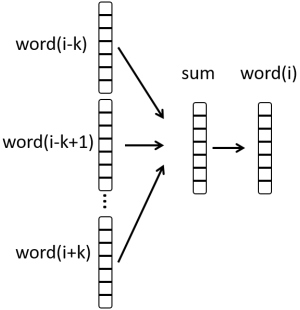
Mikolov et al. (2013a,b)

- Two models for word vectors designed to be computationally efficient
  - Continuous bag of words (CBOW): $P(v|\mathcal{C}_w)$
    - Similar in spirit to the feedforward neural language model we saw before (Bengio et al. (2003))
  - Skip-gram: : $P(\mathcal{C}_w|v)$
- It turns out these are closely related to matrix factorization as in LSI/A (Levy and Goldberg (2014))

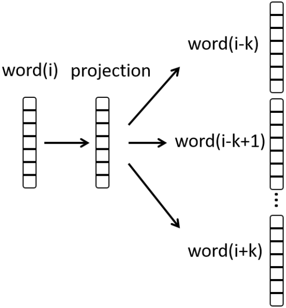
Word2vec: Overview of Two Models

Mikolov et al. (2013a,b)

- Continuous bag of words (CBOW): $P(v|\mathcal{C}_w)$
  - Use the context words (average) to predict the center word
- Skip-gram: : $P(\mathcal{C}_w|v)$
  - Use the center word to predict each of the context words



(a) CBOW



(b) Skipgram

CBOW

Skipgram can be derived similarly

- Given a sequence of training words $w_1, w_2, \dots, w_N$ in $\mathcal{W}$, the training objective is to maximize the average negative log-likelihood function

$$\mathcal{J}_{CBOW} = -\sum_{w \in \mathcal{W}} \log P(w|\mathcal{C}_w)$$

$$= -\sum_{w \in \mathcal{W}} \log \frac{\exp(\mathbf{v}_w^\top \mathbf{h}_c)}{\sum_{w' \in \mathcal{V}} \exp(\mathbf{v}_{w'}^\top \mathbf{h}_c)}$$

  where $\mathbf{h}_c = \frac{1}{|\mathcal{C}_w|}\sum_{c \in \mathcal{C}_w} \mathbf{u}_c$, $\mathcal{C}_w$ contains all words shown in a small window around w
  - For all $w, c \in \mathcal{V}$, $\mathbf{v}_w$ (parameters) and $\mathbf{u}_c$ (word embedding) are two sets of vectors
- When performing SGD to this cost function
  - Non-convex: optimize $\mathbf{v}_w$ and $\mathbf{u}_c$ simultaneously
  - Inefficient to compute $\sum_{c \in \mathcal{V}} \exp(\mathbf{v}_c^\top \mathbf{h}_c)$ for large vocabulary $\mathcal{V}$

Overview

How to Improve Efficiency?

Approach 1: a surrogate loss, hierarchical softmax

- Class based model was first proposed by Goodman (2001)

$$P(w|\mathcal{C}_w) = \sum_l P(w, class(w) = l|\mathcal{C}_w) = P(w, class(w) = l|\mathcal{C}_w)$$

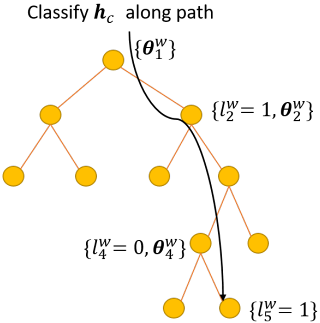
  since only one class label l is compilable with the hard clustering. So

$$P(w|\mathcal{C}_w) = P(w|class(w) = l, \mathcal{C}_w)P(class(w) = l|\mathcal{C}_w)$$

- Although any $l(\cdot)$ would yield correct probabilities, generalization could be better for choices of word classes that "make sense," i.e., those for which it easier to learn the $P(class(w) = l|\mathcal{C}_w)$ (Morin and Bengio (2005))

A generalization of class based model is to apply it for a tree of words, using

- l_i^w label of i-th node in path, $i \in \{2, \ldots, L^w\}$
- $\boldsymbol{\theta}_i^w$ as the vector representation of the i-th node in path



If we use a binary tree $l_i^w \in \{-1, 1\}$, then the classification sequence along a path consists of a sequence of logistic regression:

$$P(w|\mathcal{C}_w) = \sum_{i=2}^{L^w} P(l_i^w|\mathbf{h}_c, \boldsymbol{\theta}_{i-1}^w) = \sum_{i=2}^{L^w} \sigma(l_i^w \mathbf{h}_c^\top \boldsymbol{\theta}_{i-1}^w)$$

where $\sigma(x) = 1/(1 + \exp(-x))$

Approach 1: Hierarchical Softmax (Cont'd)

Then the objective function can be written as:

$$\mathcal{J}_{CBOW}^{HS} = - \sum_{w \in \mathcal{W}} \sum_{i=2}^{L^w} \log[\sigma(l_i^w \mathbf{h}_c^\top \boldsymbol{\theta}_{i-1}^w)]$$

Options to build the tree of words

- Wordnet (Morin and Bengio (2005))
- Hierarchical clustering (Mnih and Hinton (2008))
- Huffman tree based on word frequrencies (Mikolov et al. (2013a,b))
  - Complexity reduced from V to $\log_2 V$
  - If consider lower-frequency words are deeper, the practical performance is further improved (higher frequency words are accessed more frequently)

Hierarchical Softmax: Optimization

- For each word w at position i along the path, we denote

$$\mathcal{J}_{CBOW}^{HS}(w,i) = -\log[\sigma(l_i^w \mathbf{h}_c^\top \boldsymbol{\theta}_{i-1}^w)] = \log[1+\exp(-l_i^w \mathbf{h}_c^\top \boldsymbol{\theta}_{i-1}^w)]$$

- So we have:

$$\frac{\partial \mathcal{J}_{CBOW}^{HS}(w,i)}{\partial \boldsymbol{\theta}_{i-1}^w} = -l_i^w \sigma(-l_i^w \mathbf{h}_c^\top \boldsymbol{\theta}_{i-1}^w)\mathbf{h}_c$$

- So we have SGD for $\boldsymbol{\theta}_{i-1}^w$

$${\boldsymbol{\theta}_{i-1}^w}^{(t+1)} = {\boldsymbol{\theta}_{i-1}^w}^{(t)} + \eta l_i^w \sigma(-l_i^w \mathbf{h}_c^\top \boldsymbol{\theta}_{i-1}^w)\mathbf{h}_c$$

Hierarchical Softmax: Optimization

- Similarly, we have:

$$\frac{\partial \mathcal{J}^{HS}_{CBOW}(w,i)}{\partial \mathbf{h}_c} = -l_i^w \sigma(-l_i^w \mathbf{h}_c^\top \boldsymbol{\theta}_{i-1}^w)\boldsymbol{\theta}_{i-1}^w$$

which leads to

$$\mathbf{u}_c^{(t+1)} = \mathbf{u}_c^{(t)} + \eta \sum_{i=2}^{L^w} \frac{1}{|\mathcal{C}_w|} l_i^w \sigma(-l_i^w \mathbf{h}_c^\top \boldsymbol{\theta}_{i-1}^w)\boldsymbol{\theta}_{i-1}^w$$

since $\mathbf{h}_c = \frac{1}{|\mathcal{C}_w|}\sum_{c\in\mathcal{C}_w} \mathbf{u}_c$ and $\mathcal{C}_w$ contains all words shown in a small window around w

Overview

How to Improve Efficiency?

Approach 2: transforming the computationally expensive learning problem into a binary classification *proxy problem* that uses the same parameters but requires statistics that are easier to compute

- Recall the CBOW objective is

$$\mathcal{J}_{CBOW} = -\sum_{w \in \mathcal{W}} \log P(w|\mathcal{C}_w)$$
$$= -\sum_{w \in \mathcal{W}} \log \frac{\exp(\mathbf{v}_w^\top \mathbf{h}_c)}{\sum_{c \in \mathcal{V}} \exp(\mathbf{v}_{w'}^\top \mathbf{h}_c)}$$

  where $\mathbf{h}_c = \frac{1}{|\mathcal{C}_w|}\sum_{c \in \mathcal{C}_w} \mathbf{u}_c$, $\mathcal{C}_w$ contains all words shown in a small window around w
- We simplify the probability to be $P(w|\mathcal{C}_w) = P_\theta(w|c)$
- We denote the parameters as $\theta = \{\mathbf{v}_w, w \in \mathcal{V}\}$

Noise Contrastive Estimation (NCE)

- We denote the empirical distributions as $\tilde{P}(w|c)$ and $\tilde{P}(c)$
- We use the parameterized distribution $P_\theta(w|c)$ to approximate $\tilde{P}(w|c)$
- To avoid costly summations, a "noise" distribution, $Q(w)$, is used
  - In practice Q is a uniform, empirical unigram, or flattened empirical unigram distribution
- NCE reduces the estimation problem to the problem of estimating the parameters of a probabilistic binary classifier that
  - uses the same parameters to distinguish samples from the empirical distribution from samples generated by the noise distribution

$$P(d, w|c) = \begin{cases} \frac{k}{k+1} Q(w) & \text{if } d = 0 \\ \frac{1}{k+1} \tilde{P}(w|c) & \text{if } d = 1 \end{cases}$$

1. Sample a c from $\tilde{P}(c)$ and given c
2. Sample a w from $\tilde{P}(w|c)$ (true distribution) and k of w from $Q(w)$ (noise)

Noise Contrastive Estimation (NCE) (Cont'd)

- From the joint conditional probability

$$P(d, w|c) = \begin{cases} \frac{k}{k+1} Q(w) & \text{if } d = 0 \\ \frac{1}{k+1} \tilde{P}(w|c) & \text{if } d = 1 \end{cases}$$

  - Using definition of conditional probability

$$P(d|c, w) = \begin{cases} \frac{\frac{k}{k+1} Q(w)}{\frac{k}{k+1} Q(w) + \frac{1}{k+1} \tilde{P}(w|c)} = \frac{kQ(w)}{\tilde{P}(w|c) + kQ(w)} & \text{if } d = 0 \\ \frac{\tilde{P}(w|c)}{\tilde{P}(w|c) + kQ(w)} & \text{if } d = 1 \end{cases}$$

- NCE replaces the empirical distribution $\tilde{P}(w|c)$ with the model distribution $P_\theta(w|c)$, and θ is chosen to maximize the conditional likelihood of the "proxy corpus" created as described above

Noise Contrastive Estimation (NCE) (Cont'd)

- So we have

$$P_\theta(d|c,w) = \begin{cases} \frac{kQ(w)}{P_\theta(w|c)+kQ(w)} & \text{if } d=0 \\ \frac{P_\theta(w|c)}{P_\theta(w|c)+kQ(w)} & \text{if } d=1 \end{cases}$$

- Recall the original (simplified) negative log likelihood is

$$\mathcal{J}_{CBOW} = -\sum_{w\in\mathcal{W}} \log P_\theta(w|c) = -\mathbb{E}_{\tilde{P}(w|c)} \log P_\theta(w|c)$$

- With NCE, θ can be trained to maximize the expectation of $\log P_\theta(d|c,w)$ under the mixture of the data and noise samples

$$\mathcal{J}_{NCE_k} = -\mathbb{E}_{\tilde{P}(w|c)}\left[\log \frac{P_\theta(w|c)}{P_\theta(w|c)+kQ(w)}\right] - k\mathbb{E}_{Q(w)}\left[\log \frac{kQ(w)}{P_\theta(w|c)+kQ(w)}\right]$$

Asymptotic Property

- Given that

$$\mathcal{J}_{NCE_k} = -\mathbb{E}_{\tilde{P}(w|c)}\left[\log \frac{P_\theta(w|c)}{P_\theta(w|c)+kQ(w)}\right] - k\mathbb{E}_{Q(w)}\left[\log \frac{kQ(w)}{P_\theta(w|c)+kQ(w)}\right]$$

- The gradient is

$$\begin{aligned}\frac{\partial \mathcal{J}_{NCE_k}}{\partial\theta} =& -\mathbb{E}_{\tilde{P}(w|c)}\left[\frac{kQ(w)}{P_\theta(w|c)+kQ(w)}\frac{\partial}{\partial\theta}\log P_\theta(w|c)\right] \\ &+ k\mathbb{E}_{Q(w)}\left[\frac{P_\theta(w|c)}{P_\theta(w|c)+kQ(w)}\frac{\partial}{\partial\theta}\log P_\theta(w|c)\right] \\ =& -\sum_w \frac{kQ(w)}{P_\theta(w|c)+kQ(w)}\left[\tilde{P}(w|c) - P_\theta(w|c)\right]\frac{\partial}{\partial\theta}\log P_\theta(w|c)\end{aligned}$$

- As $k \rightarrow \infty$, we have

$$\frac{\partial \mathcal{J}_{NCE_k}}{\partial\theta} \rightarrow -\sum_w \left[\tilde{P}(w|c) - P_\theta(w|c)\right]\frac{\partial}{\partial\theta}\log P_\theta(w|c)$$

which is the maximum likelihood gradient (the gradient is 0 when the model distribution matches the empirical distribution)

Practical Issues

- In practice, we do random sampling to generate k noise samples to perform estimation

$$
\begin{aligned}
\mathcal{J}_{NCE_k} &= -\sum_{w,c\in\mathcal{D}} [\log P(d=1|c,w) + k\mathbb{E}_{w'\sim Q(w)} \log P(d=0|c,w')] \\
&\approx -\sum_{w,c\in\mathcal{D}} [\log P(d=1|c,w) + k \sum_{i=1,w'\sim Q(w)}^{k} \frac{1}{k} \log P(d=0|c,w')] \\
&= -\sum_{w,c\in\mathcal{D}} [\log P(d=1|c,w) + \sum_{i=1,w'\sim Q(w)}^{k} \log P(d=0|c,w')]
\end{aligned}
$$

- Then the stochastic gradient is

$$
\begin{aligned}
\frac{\partial \mathcal{J}^{w}_{NCE_k}}{\partial\theta} = -[&\frac{kQ(w)}{P_\theta(w|c)+kQ(w)} \frac{\partial}{\partial\theta} \log P_\theta(w|c) \\
&- \sum_{i=1,w'\sim Q(w)}^{k} \frac{P_\theta(w'|c)}{P_\theta(w'|c)+kQ(w')} \frac{\partial}{\partial\theta} \log P_\theta(w'|c)]
\end{aligned}
$$

Practical Issues

- NCE replaces the empirical distribution $\tilde{P}(w|c)$ with the model distribution $P_\theta(w|c)$
  - But $P_\theta(w|c) = \frac{f_\theta(w,c)}{\sum_{w'} f_\theta(w',c)} = \frac{f_\theta(w,c)}{Z_\theta(c)}$ still requires evaluating the partition function $Z_\theta(c)$
  - Original NCE introduce a parameter to estimate $Z_\theta(c)$ for every possible c, which is still huge in language models (Mnih and Teh (2012))
    - This approach is, however, not possible for Maximum Likelihood Estimation since the likelihood can be made arbitrarily large by making Z go to zero.(Gutmann and Hyvärinen (2012))
  - For neural networks, original paper simply set $Z_\theta(c) = 1$ (Mnih and Teh (2012)), which result in

$$P(d|c,w) = \begin{cases} \frac{kQ(w)}{f_\theta(w,c)+kQ(w)} & \text{if } d = 0 \\ \frac{f_\theta(w,c)}{f_\theta(w,c)+kQ(w)} & \text{if } d = 1 \end{cases}$$

  and claimed they found comparable results
  - Intuitively, by parameterizing $\log P_\theta(w,c)$, $\log Z_\theta$ can be considered as a bias term in addtition to the parameters of $\log f_\theta(w,c)$

Negative Sampling for Word2vec

- Now we derive negative sampling for word2vec and compare with general NCE strategy
- The proposed proxy distribution of negative sampling is

$$P(d|c,w) = \begin{cases} \frac{1}{f_\theta(w,c)+1} & \text{if } d=0 \\ \frac{f_\theta(w,c)}{f_\theta(w,c)+1} & \text{if } d=1 \end{cases}$$

Compared to NCE:

$$P(d|c,w) = \begin{cases} \frac{kQ(w)}{f_\theta(w,c)+kQ(w)} & \text{if } d=0 \\ \frac{f_\theta(w,c)}{f_\theta(w,c)+kQ(w)} & \text{if } d=1 \end{cases}$$

  - Negative sampling is equivalent to NCE when $k = |\mathcal{V}|$ and $Q(w)$ is uniform
  - Aside from the $k = |\mathcal{V}|$ and uniform $Q(w)$ case, the conditional probabilities of d given (w,c) are not consistent with the language model probabilities of $P_\theta(w|c)$
    - It does not have the same asymptotic consistency guarantees that NCE has

Negative Sampling for CBOW

- Recall the CBOW objective is

$$\mathcal{J}_{CBOW} = -\sum_{w \in \mathcal{W}} \log P(w|\mathcal{C}_w) = -\sum_{w \in \mathcal{W}} \log \frac{\exp(\mathbf{v}_w^\top \mathbf{h}_c)}{\sum_{w' \in \mathcal{V}} \exp(\mathbf{v}_{w'}^\top \mathbf{h}_c)}$$

  where $\mathbf{h}_c = \frac{1}{|\mathcal{C}_w|} \sum_{c \in \mathcal{C}_w} \mathbf{u}_c$, $\mathcal{C}_w$ contains all words shown in a small window around w
- By introducing the proxy distribution:

$$P(d|c, w) = \begin{cases} \frac{1}{f_\theta(w,c)+1} & \text{if } d = 0 \\ \frac{f_\theta(w,c)}{f_\theta(w,c)+1} & \text{if } d = 1 \end{cases}$$

- We have the following objective function for (CBOW) word embedding with negative sampling:

$$\mathcal{J}_{CBOW}^{NS} = -\sum_{w \in \mathcal{W}} [\log \frac{\exp(\mathbf{v}_w^\top \mathbf{h}_c)}{\exp(\mathbf{v}_w^\top \mathbf{h}_c) + 1} + \sum_{w' \in Q(w)}^{k} \log \frac{1}{\exp(\mathbf{v}_{w'}^\top \mathbf{h}_c) + 1}]$$

Learning with Negative Sampling

- Starting from the objective of CBOW using negative sampling

$$
\begin{aligned}
\mathcal{J}_{CBOW}^{NS} &= -\sum_{w \in \mathcal{W}} [\log \frac{\exp(\mathbf{v}_w^\top \mathbf{h}_c)}{\exp(\mathbf{v}_w^\top \mathbf{h}_c) + 1} + \sum_{w' \in Q(w)}^{k} \log \frac{1}{\exp(\mathbf{v}_{w'}^\top \mathbf{h}_c) + 1}] \\
&= -\sum_{w \in \mathcal{W}} [\log \sigma(\mathbf{v}_w^\top \mathbf{h}_c) + \sum_{w' \in Q(w)}^{k} \log \sigma(-\mathbf{v}_{w'}^\top \mathbf{h}_c)] \\
&\doteq -\sum_{u \in \mathcal{W} \cup \mathcal{N}(w)} \log \sigma(l^u \mathbf{v}_u^\top \mathbf{h}_c) \\
&= \sum_{u \in \mathcal{W} \cup \mathcal{N}(w)} \log[1 + \exp(-l^u \mathbf{v}_u^\top \mathbf{h}_c)]
\end{aligned}
$$

where $\mathcal{N}(w)$ is the set of negative sampling, l^u is a binary label:

- $l^u = 1$ represents the word is from empirical distribution and $u = w$
- $l^u = -1$ represents the word is from the proxy distribution and $u = w'$

Learning with Negative Sampling (Cont'd)

- So the gradient of $\mathcal{J}_{CBOW}^{NS}$ w.r.t. $\boldsymbol{\theta}_w = \mathbf{v}_u$ is

$$\frac{\partial \mathcal{J}_{CBOW}^{NS}}{\partial \mathbf{v}_u} = -l^u \sigma(-l^u \mathbf{v}_u^\top \mathbf{h}_c)\mathbf{h}_c$$

  and w.r.t. $\mathbf{h}_c$ is

$$\frac{\partial \mathcal{J}_{CBOW}^{NS}}{\partial \mathbf{h}_c} = -l^u \sigma(-l^u \mathbf{v}_u^\top \mathbf{h}_c)\mathbf{v}_u$$

- So SGD for $\boldsymbol{\theta}_w = \mathbf{v}_u$ is

$$\mathbf{v}_u^{t+1} = \mathbf{v}_u^t + \eta l^u \sigma(-l^u \mathbf{v}_u^\top \mathbf{h}_c)\mathbf{h}_c$$

- and SGD for $\mathbf{u}_c$ is

$$\mathbf{u}_c^{t+1} = \mathbf{u}_c^t + \eta \frac{1}{|\mathcal{C}_w|} l^u \sigma(-l^u \mathbf{v}_u^\top \mathbf{h}_c)\mathbf{v}_u$$

Summary of Negative Sampling

- NCE is an effective way of learning parameters for an arbitrary locally normalized language model
- Negative sampling should be thought of as an alternative task for generating representations of words for use in other tasks
  - It is not a method for learning parameters in a generative model of language

- If your goal is language modeling, you should use NCE
- If your goal is word representation learning, you should consider both NCE and negative sampling

Overview

Word Vector Evaluations

See `http://wordvectors.org` for a suite of examples.

- Several popular methods for *intrinsic* evaluations:
  - Do (cosine) similarities of pairs of words' vectors correlate with judgments of similarity by humans?
  - TOEFL-like synonym tests, e.g., rug $\rightarrow$ {sofa, ottoman, carpet, hallway}
  - Syntactic analogies, e.g., "walking is to walked as eating is to what?" Solved via:

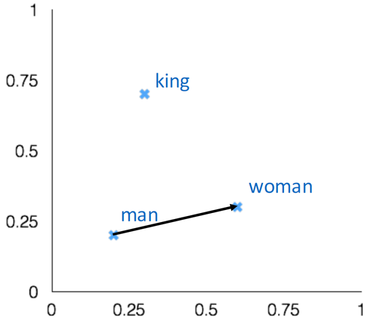
$$\min_{\mathbf{v} \in \mathcal{V}} \cos(\mathbf{v}_v, \mathbf{v}_{walking} - \mathbf{v}_{walked} + \mathbf{v}_{eating})$$

- Also: *extrinsic* evaluations on NLP tasks that can use word vectors (e.g., sentiment analysis)

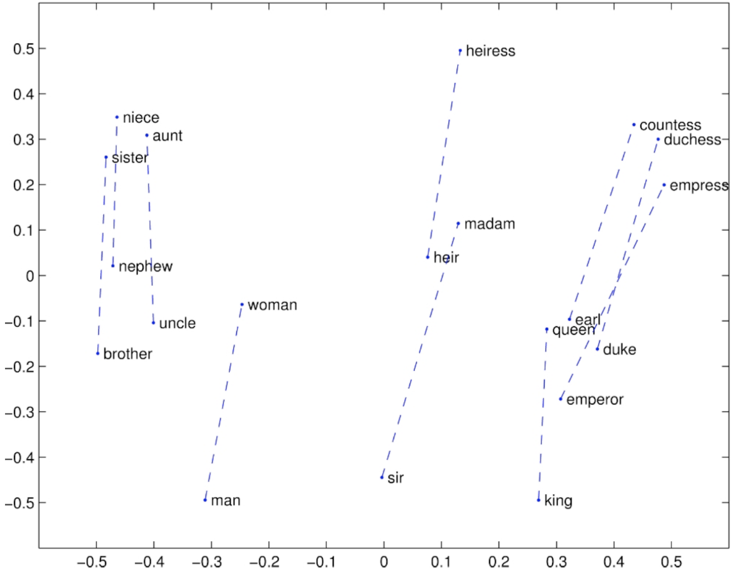
Word Analogy

- Evaluate word vectors by how well their cosine distance after addition captures intuitive semantic and syntactic analogy questions
- Discarding the input words from the search!
- Problem: What if the information is there but not linear?

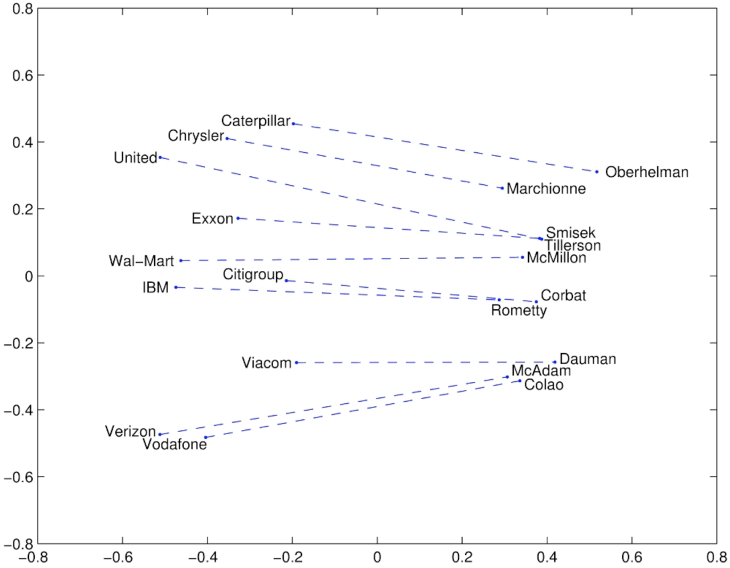
$$\min_{\mathbf{v}\in\mathcal{V}} \cos(\mathbf{v}_v, \mathbf{v}_{man} - \mathbf{v}_{womon} + \mathbf{v}_{king})$$



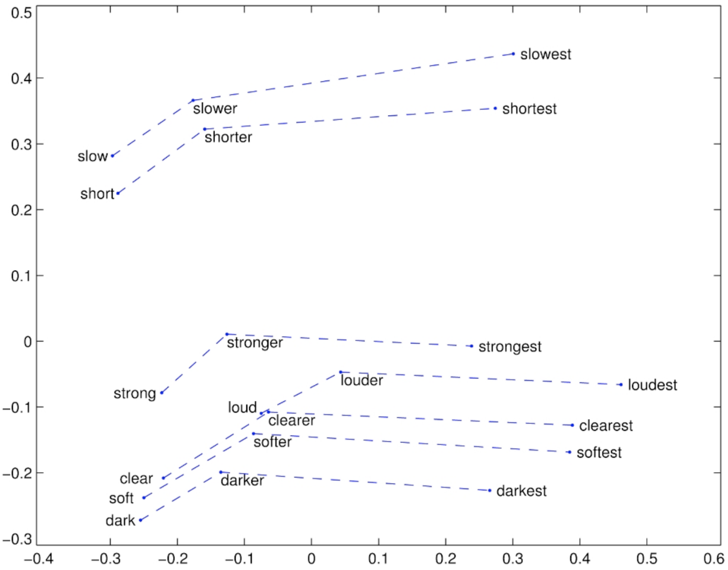
Word Analogy: Glove (Pennington et al. (2014)



Glove Visualizations: Company - CEO

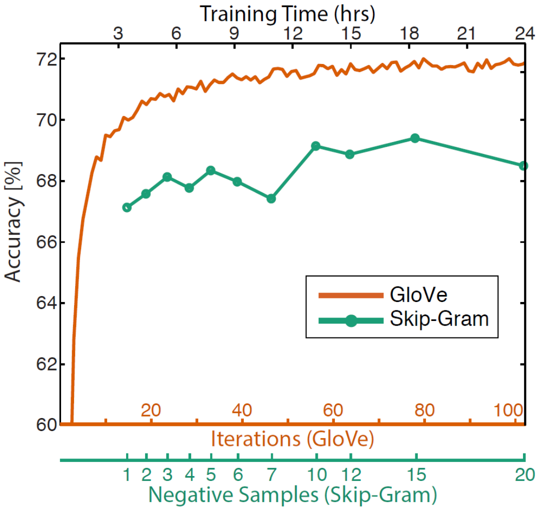


Glove Visualizations: Superlatives



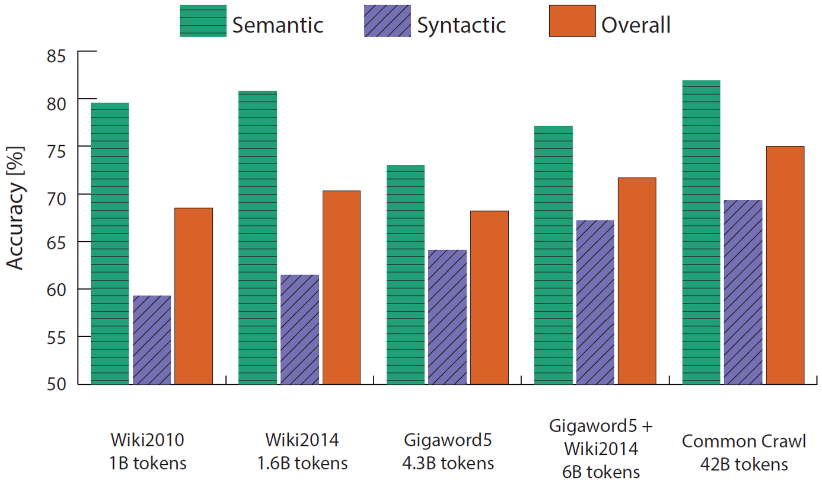
Analogy Evaluation and Hyperparameters

- More training time helps



Analogy Evaluation and Hyperparameters

- More data helps, Wikipedia is better than news text!



Arguments from Yoav GoldBurg

http://u.cs.biu.ac.il/~yogo/cvsc2015.pdf

- Nothing magical about embeddings
- It is just the same old distributional word similarity in a shiny new dress
- "But word2vec is still better, isn't it?"
  - Plenty of evidence that word2vec outperforms traditional methods (In particular: "Don't count, predict!" (Baroni et al. (2014))
  - How does this fit with our story?
- The Big Impact of "Small" Hyperparameters
  - word2vec is more than just an algorithm
  - Introduces many engineering tweaks and hyperpararameter settings
    - May seem minor, but make a big difference in practice
    - Their impact is often more significant than the embedding algorithm's
  - These modifications can be ported to distributional methods

Arguments from Yoav GoldBurg

http://u.cs.biu.ac.il/~yogo/cvsc2015.pdf

- There is no single downstream task
  - Different tasks require different kinds of similarity
  - Different vector-inducing algorithms produce different similarity functions
  - No single representation for all tasks
- "but my algorithm works great for all these different word-similarity datasets! doesn't it mean something?"
  - Sure it does
  - It means these datasets are not diverse enough
  - They should have been a single dataset
  - (alternatively: our evaluation metrics are not discriminating enough)

Further Reading

- Potts (2013). Distributional approaches to word meanings. Ling 236/Psych 236c: Representations of meaning, Spring 2013
- Goldberg and Levy (2014). word2vec Explained: deriving Mikolov et al.'s negative-sampling word-embedding method
- Dyer (2014). Notes on Noise Contrastive Estimation and Negative Sampling.

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